the skill with which the author covers them. The list of 113 references is also outstanding.

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16[26–02, 26D15, 26B25].—P. S. BULLEN, D. S. MITRINOVIĆ & P. M. VASIĆ, *Means and Their Inequalities*, Mathematics and Its Applications (East European Series), Kluwer, Dordrecht, 1988, xix + 459 pp., $24\frac{1}{2}$ cm. Price $89.00.

A mean of the positive values $a_1, \ldots, a_n$ is a function $F(a_1, \ldots, a_n)$ whose value lies between the smallest and largest of its arguments. Weighted means depend also on positive weights $w_1, \ldots, w_n$ with $\sum w_i = 1$. By far the most important means are the arithmetic and geometric means, $\sum w_i a_i$ and $\prod a_i^{w_i}$. These two, together with the harmonic mean $(\sum w_i a_i^{-1})^{-1}$, occupy nearly a quarter of the book under review. It is a very comprehensive survey of nearly everything that has been published on mean values, as illustrated by 52 proofs of the fundamental inequality stating that the geometric mean does not exceed the arithmetic mean. The bibliography fills 63 pages and contains approximately a thousand entries, clear evidence that the current revision of the Mathematics Subject Classification needs a better pigeonhole for means than the 1985 version provides.

Almost another quarter of the book is occupied by the power mean $(\sum w_i a_i^r)^{1/r}$, which includes the geometric mean when $r \to 0$ as well as the arithmetic and harmonic means. Its increase with r generalizes the inequality of arithmetic and geometric means, and its other properties include the inequalities of Cauchy, Hölder, and Minkowski. While the power mean is homogeneous in $a_1, \ldots, a_n$, a further generalization called the quasi-arithmetic mean, $\phi^{-1}[\sum w_i \phi(a_i)]$ with ϕ continuous and strictly monotonic, is not homogeneous unless $\phi(x)$ is a linear function of x^r or $\log x$. Other main topics are symmetric means like $[(a_1a_2 + a_1a_3 + a_2a_3)/3]^{1/2}$, means constructed in various esoteric ways, iterated means like Gauss's arithmetic-geometric mean, and finally integral means of functions. Means of operators are mentioned but not discussed. There is a list of notations and symbols and an index of authors but no subject index.

For anyone doing research on mean values or looking for an inequality between means, the book is a splendid reference work in spite of many misprints, a photographically reduced typescript with small characters, and the absence of boldface or large headings that would make it easier to navigate. One or more proofs of nearly every theorem are given expertly in a consistent notation with careful attention to conditions of equality. Above all, the reader will find many references to published papers that he would be very unlikely to discover otherwise. Because applications to other branches of mathematics are rarely mentioned, he may get the impression that inequalities for mean values have become a somewhat ingrown field of research without a strong sense of future directions. This impression will perhaps be proved incorrect by a complementary volume of applications, comments, and further results, now being prepared by the senior author (D.S.M.) and two collaborators.

B. C. C.

17[65–06, 65D30, 65D32].—H. BRASS & G. H. HÄMMERLIN (Editors), *Numerical Integration III*, International Series of Numerical Mathematics, Vol. 85, Birkhäuser, Basel, 1988, xiv + 325 pp., 24 cm. Price $60.50.

These are the proceedings of the third conference on numerical integration held at the Oberwolfach Mathematics Research Institute November 8–14, 1987. (The proceedings of the 1978 and 1981 conferences were published in Volumes 45 and 57 of the same series.) There are 28 papers, about three quarters of which deal with one-dimensional integration. The great variety of topics addressed during this conference can be gathered from the following list of key words: Computation of convolution integrals, Stieltjes integrals, and principal value integrals; Gauss and Chebyshev type quadrature rules; optimal quadrature; product integration; positivity of interpolatory rules; error estimation and convergence acceleration; theorems of Bernstein-Jackson type; cubature formulae with minimal or almost minimal number of knots; criteria for constructing multidimensional integration rules; quasi-Monte Carlo methods; lattice rules. The volume concludes with a traditional section on open problems.

W. G.

18[11A41, 11Y05, 11Y11].—HIDEO WADA, *Computers and Prime Factorization* (Japanese), Yūsei Publishers, Tōkyō, 1987, 190 pp., 21 cm. Price Yen 1800.

Most methods of obtaining the prime factorization of a given natural number have two steps. In the first step one decides whether the integer is prime or composite. In the second step one finds a nontrivial factor of the integer, if it is composite. The complete prime factorization is produced by performing the two steps recursively while composite factors remain. This book is an introduction to modern algorithms for these two steps.

To decide whether a large integer n is prime or not, one often checks whether it satisfies the conclusion of Fermat's Little Theorem, $b^{n-1} \equiv 1 \pmod{n}$, for some b. If this congruence fails and n is relatively prime to b, then n is definitely composite and we try to factor it. If the congruence holds, then n is almost certain to be prime and we use a *prime-proving algorithm* to show rigorously that n is prime. Rigorous tests for primeness are still much slower than probabilistic ones.

The book begins with some preliminaries from elementary number theory: Euclid's algorithm, congruences and Euler's totient function. The first algorithm after Euclid's is trial division, the only algorithm which factors ånd proves primality as well (for small integers). The next algorithm, fast modular exponentiation, is used